TRINITY COLLEGE

Year 12 Mathematics Specialist Units 3, 4
Test 4 2020

Section 1 Calculator Free
Integration and Applications of Integration

STUDENT'S NAME ____

DATE: Monday 27 July **TIME:** 33 minutes **MARKS**: 33

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser, Formula Booklet

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (4 marks)

Determine the following integrals:

(a) $\int \frac{x-1}{x}\,dx$ [2]

(b) $\int x\cos(x^2)\,dx$ [2]

2. (9 marks)

Determine the following integrals:

(a) $\int \frac{\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\cos\theta+1}\,d\theta$ [3]

(b) $\int \cos^3 x\,dx$ [3]

(c) $\int \frac{2x^2}{x+1}\,dx$ [3]

3. (6 marks)

(a) Express $\dfrac{\frac{7}{2}-x}{(x-1)(2x+3)}$ in the form $\dfrac{a}{x-1}+\dfrac{b}{2x+3}$. [3]

(b) Hence, determine $\displaystyle\int \frac{\frac{7}{2}-x}{(x-1)(2x+3)}\,dx$ [3]

4. (5 marks)

Evaluate exactly: $\int_0^1 \frac{1}{\sqrt{4-x^2}}\,dx$ using the substitution $x = 2\sin\theta$

5. (9 marks)

Consider the integrals $I = \int_0^a \frac{f(x)}{f(x)+f(a-x)}\,dx$ and $J = \int_0^a \frac{f(a-x)}{f(x)+f(a-x)}\,dx$

(a) Use the substitution $u = a - x$ to show that $I = J$. [3]

(b) By considering $I + J$, or otherwise, evaluate I in terms of a . [2]

(c) Use the result from (b) and $\cos\theta = \sin(\frac{\pi}{2} - \theta)$ to evaluate $\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin(x + \frac{\pi}{4})} \, dx$. [4]

TRINITY COLLEGE

Year 12 Mathematics Specialist Units 3, 4
Test 4 2020

Section 2 Calculator Assumed

Integration and Applications of Integration

STUDENT'S NAME ____

DATE: Monday 27 July **TIME:** 17 minutes **MARKS**: 16

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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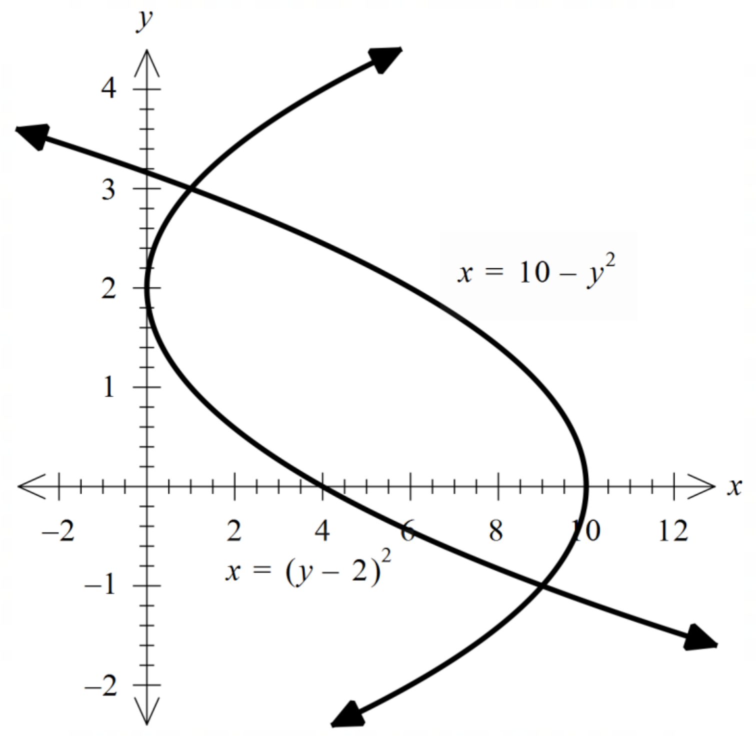
6. (2 marks)

Evaluate $\int_{-1}^{1} e^{-x^2}\,dx$ to 2 decimal places.

7. (8 marks)

Consider the two curves below.



(a) (i) Write an integral expression for the enclosed area between the curves. [2]

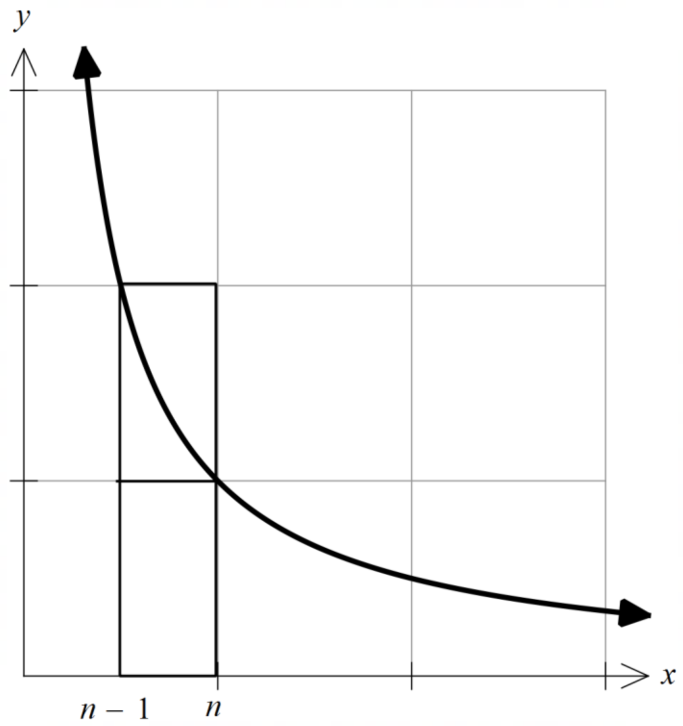
(ii) Calculate the enclosed area. [2]

(b) (i) Write down an integral expression for volume formed when the enclosed region is rotated about the y-axis. [2]

(ii) Calculate the volume formed when the enclosed region is rotated about the y-axis. [2]

8. (6 marks)

Let n be a positive integer greater than 1. The area of the region under the curve $y = \frac{1}{x}$ from $x = n-1$ to $x = n$ lies between the areas of the two rectangles, as shown in the diagram.



(a) Determine an expression for the area of the larger rectangle. [1]

(b) Use the diagram to show that the area under the curve between $n-1$ and n satisfies

$$\frac{1}{n} < \ln\left(\frac{n}{n-1}\right) < \frac{1}{n-1}$$ [2]

(c) Use the result from (b) to show that $e^{\frac{-n}{(n-1)}} < \left(1 - \frac{1}{n}\right)^n < e^{-1}$ [3]